Name: Key

1) This question is about gravitational fields.

(a) Define *gravitational field strength*.

The force exerted per unit mass on a point or small mass OR $g = \frac{F}{m}$

(2)

The gravitational field strength at the surface of Jupiter is 25 N kg^{-1} and the radius of Jupiter is 7.1×10^7 m.

(b) (i) Derive an expression for the gravitational field strength at the surface of a planet in terms of its mass *M*, its radius *R* and the gravitational constant *G*.

Since $F_g = mg$ and $F_g = \frac{GMm}{R^2}$

$mg = \frac{GMm}{R^2} \Rightarrow g = \frac{GM}{R^2}$

(2)

(ii) Use your expression in (b)(i) above to estimate the mass of Jupiter.

$m = \frac{gR^2}{G} = \frac{(25 N kg^{-1})(7.1 \times 10^7 m)^2}{6.67 \times 10^{-11} N m^2 kg^{-2}}$

$= 1.9 \times 10^{27} kg$

(2)

2) This question is about gravitation.

(a) (i) Define *gravitational potential* at a point in a gravitational field. $\frac{E_p}{m} = V$

Work done per unit mass in moving a small mass from infinity to that point

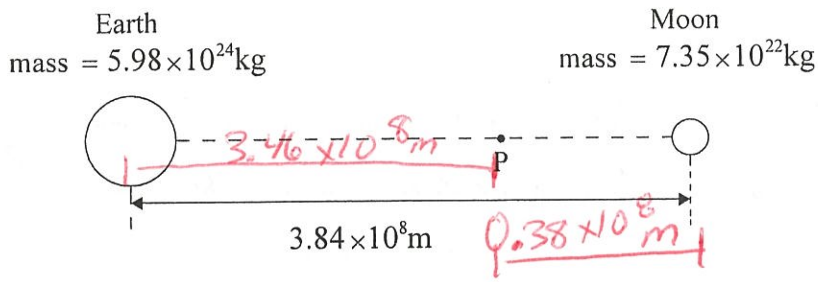
(2)

(ii) Explain why values of gravitational potential have negative values.

Since gravitational forces are always attractive the field does work when pulling a mass towards itself work done against the field is negative

(2)

The Earth and the Moon may be considered to be two isolated point masses. The masses of the Earth and the Moon are 5.98×10^{24} kg and 7.35×10^{22} kg respectively and their separation is 3.84×10^{8} m, as shown below. The diagram is not to scale.



(b) (i) Deduce that, at point P, 3.46×10^{8} m from Earth, the gravitational field strength is approximately zero.

g via Earth at P

$g = \frac{GM}{r^2}$

$= \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{3.46 \times 10^{8} m^2}$ 5.0×10^{7} 6

$= 3.33 \times 10^{-3}$ N kg or

g via Moon at P

$g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11})(7.35 \times 10^{22})}{(0.38 \times 10^{8} m)^2}$

$= 3.4 \times 10^{-3}$ N kg^{-1} OR 5.09×10^{7} 6

Field strengths are equal + opposite

(3)

(ii) The gravitational potential at P is -1.28×10^{6} J kg^{-1}. Calculate the minimum speed of a space probe at P so that it can escape from the attraction of the Earth and the Moon.

$\Delta PE = \Delta KE$ $V = \frac{\frac{1}{2}mv^2}{m} \rightarrow V = \frac{1}{2}v^2$

$V = \frac{\Delta PE}{m} = \frac{\Delta KE}{m}$

$v = \sqrt{2V}$

$= \sqrt{2(1.28 \times 10^{6})}$

$= 1.6 \times 10^{3}$ m s^{-1}

(3)

3) This question is about a spacecraft.

A spacecraft above Earth's atmosphere is moving away from the Earth. The diagram below shows two positions of the spacecraft. Position A and position B are well above Earth's atmosphere.

At position A, the rocket engine is switched off and the spacecraft begins coasting freely. At position A, the speed of the spacecraft is 5.37×10^{3} m s^{-1} and at position B, 5.10×10^{3} m s^{-1}. The time to travel from position A to position B is 6.00×10^{2} s.

(a) (i) Explain why the speed is changing between positions A and B.

It decelerates due to the gravitational pull of Earth

(1)

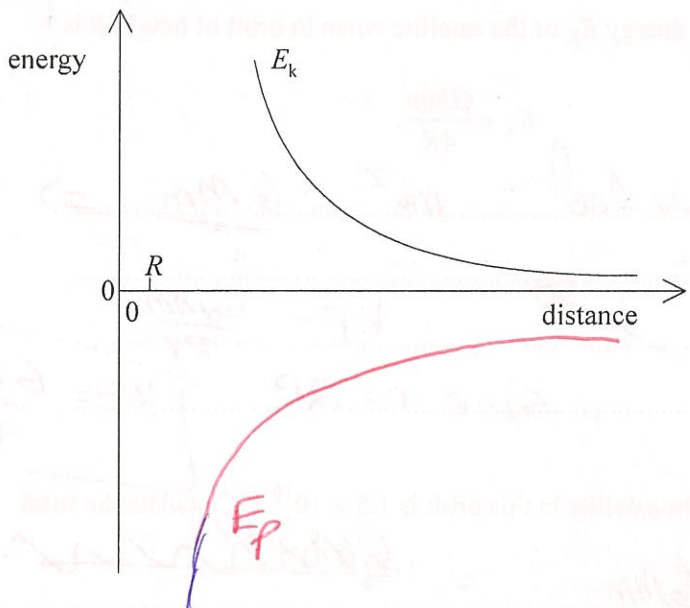
(ii) Calculate the average acceleration of the spacecraft between positions A and B.

$a = \frac{\Delta v}{t} = \frac{5100 - 5370\ ms^{-2}}{600\ s}$

$= -0,45\ ms^{-2}$

(2)

(b) The diagram below shows the variation with distance from Earth of the kinetic energy E_k of the spacecraft. The radius of Earth is R.



On the diagram above, draw the variation with distance from the surface of Earth of the gravitational potential energy E_p of the spacecraft.

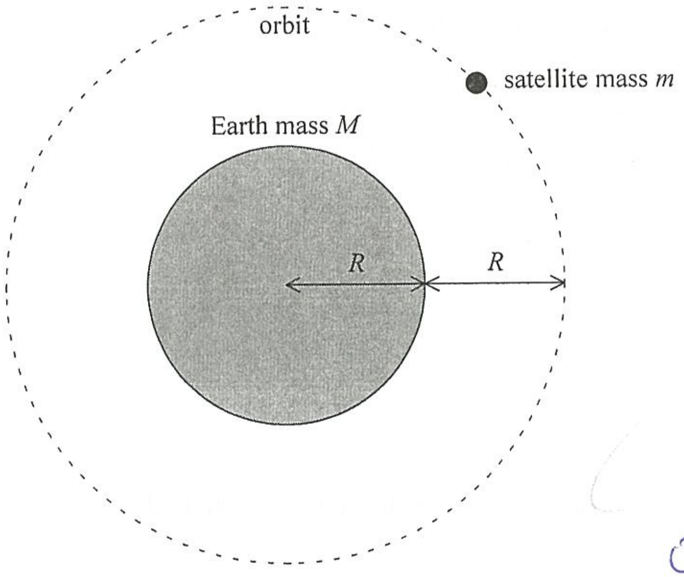
(2)

4) Motion of a satellite

(a) Define *gravitational potential.*

(2)

(b) A satellite of mass m is in a circular orbit around the Earth at height R from the Earth's surface. The mass of the Earth may be considered to be a point mass concentrated at the Earth's centre. The Earth has mass M and radius R.



OR $KE = \frac{1}{2}mv^2$

$v = \sqrt{\frac{GM}{r}}$ ←

(i) Deduce that the kinetic energy E_K of the satellite when in orbit of height R is

$$E_K = \frac{GMm}{4R}.$$

Centripetal force provided by gravitation

so

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$mv^2 = \frac{GMm}{r} \Rightarrow \frac{mv^2}{2} = \frac{GMm}{2r}$

$KE = \frac{GMm}{2r}$

Since $r = 2R$ $\boxed{KE = \frac{GMm}{4R}}$ (3)

(ii) The kinetic energy of the satellite in this orbit is 1.5×10^{10} J. Calculate the **total energy** of the satellite.

$E_T = E_K + E_P$

$= \frac{GMm}{2r} - \frac{GM}{r}$

$E_p = -\frac{GMm}{r} \Rightarrow \frac{-GMm}{2R}$

$E_T = KE + PE = \frac{GMm}{4R} - \frac{GMm}{2R} = \boxed{\frac{-GMm}{4R}} = \boxed{-1.5 \times 10^{10} J}$

(3)

(iii) Explain how your answer to (b)(ii) indicates that the satellite will not escape the Earth's gravitational field and state the minimum amount of energy that must be provided to this satellite so that it does escape.

The satellite has negative energy and cannot escape to infinity

It needs 1.5×10^{10} J of energy to escape

(3)

5) This question is about gravitational potential.

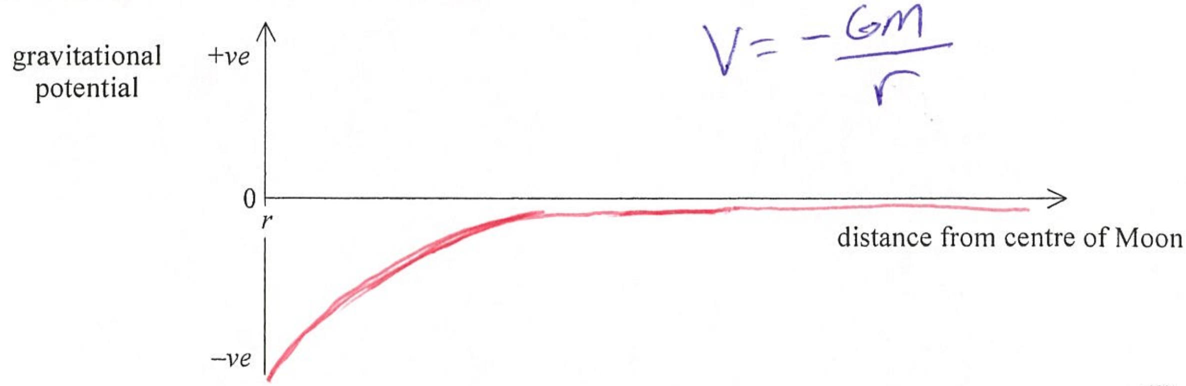
(a) Define *gravitational potential at a point.*

change in potential energy or work done per unit mass in moving small point mass from infinity to that point

(2)

(b) A meteorite moves towards the Moon from a long distance away.

(i) On the axes below, sketch a graph to show the variation with distance from the centre of the Moon of the gravitational potential of the meteorite as it approaches the Moon. The radius of the Moon is r.



$V = -\frac{GM}{r}$

(2)

(ii) The radius r of the Moon is 1.7×10^6 m and its mass is 7.3×10^{22} kg.

Estimate the impact speed with which the meteorite hits the surface of the Moon.

Change in gravitational potential $\rightarrow V = -\frac{GM}{r}$

$V = \frac{(6.67\times10^{-11})(7.3\times10^{22}\,kg)}{1.7\times10^{6}\,m} = 2.86\times10^{6}\,N/kg$

$\Delta V = \frac{\Delta E}{m} \leftarrow = KE \Rightarrow \frac{\frac{1}{2}mv^2}{m} \Rightarrow V = \frac{1}{2}v^2$

$v^2 = 2V \Rightarrow v = \sqrt{2V} = \sqrt{2(2.86\times10^{6})}$

$= 2.4\times10^{3}\,ms^{-1}$

(3)

(iii) Suggest **one** factor that will make the impact speed greater than your estimate.

meteorite may have an initial velocity towards the moon

(1)

(c) A similar meteorite moves towards the Earth from a long distance away.

Suggest how the **total** energy of the meteorite varies with distance when the meteorite is

(i) outside the Earth's atmosphere;

Constant → no frictional forces

....

(1)

(ii) inside the Earth's atmosphere.

Decreasing → more negative

....

(1)
(Total 10 marks)