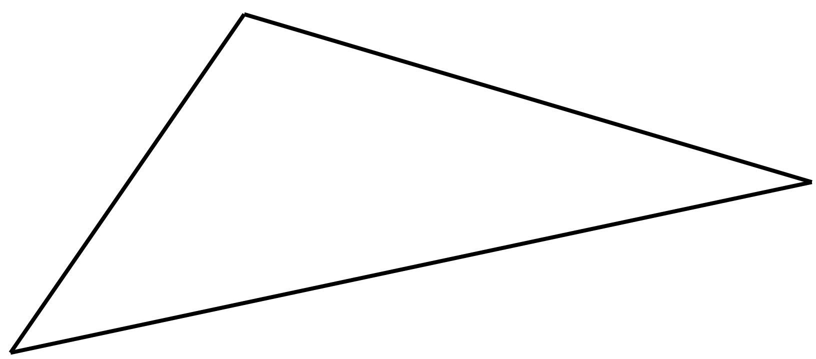
Lesson 5: Points of Concurrencies

Classwork

Opening Exercise

You will need a make-shift compass made from string and pencil

Use these materials to construct the perpendicular bisectors of the three sides of the triangle below (like you did with Problem Set # 2).



How did using this tool differ from using a compass and straightedge? Compare your construction with that of your partner. Did you obtain the same results?

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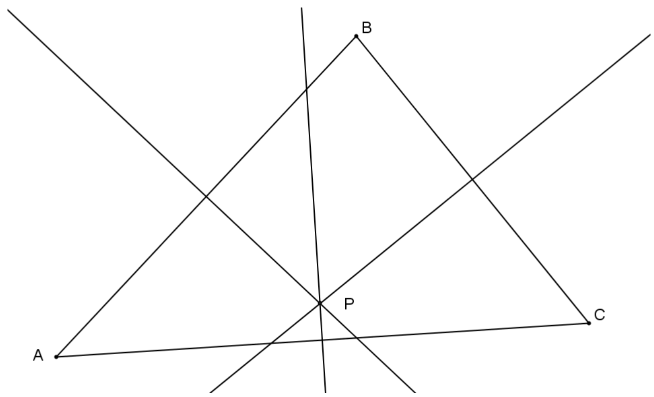
Discussion

When three or more lines intersect in a single point, they are _____________________, and the point of intersection is the _____________________________.

You saw an example of a point of concurrency in yesterday's problem set (and in the Opening Exercise above) when all three perpendicular bisectors passed through a common point.

The point of concurrency of the three perpendicular bisectors is the ____.

The circumcenter of $\triangle ABC$ is shown below as point P.



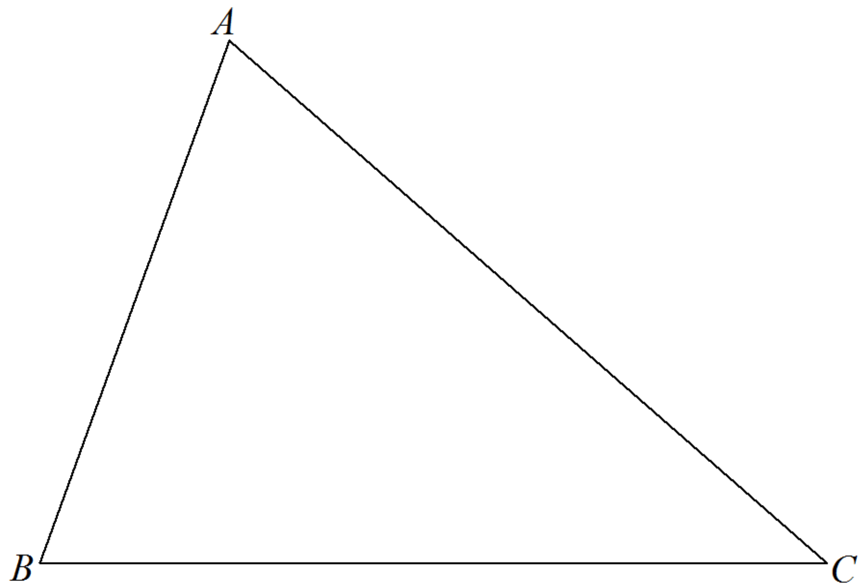
The question that arises here is: WHY are the three perpendicular bisectors concurrent? Will these bisectors be concurrent in all triangles? To answer these questions, we must recall that all points on the perpendicular bisector are equidistant from the endpoints of the segment. This allows the following reasoning:

1. P is equidistant from A and B since it lies on the ____ of AB.
2. P is also ____ from B and C since it lies on the perpendicular bisector of BC.
3. Therefore, P must also be equidistant from A and C.

Hence, $AP = BP = CP$, which suggests that P is the point of ____________________ of all three perpendicular bisectors.

You have also worked with angles bisectors. The construction of the three angle bisectors of a triangle also results in a point of concurrency, which we call the __________________________________.

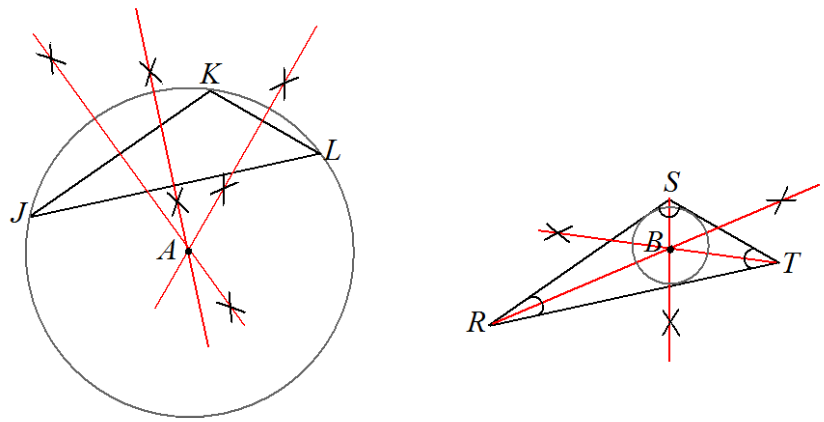
Use the triangle below to construct the angle bisectors of each angle in the triangle to locate the triangle's incenter.



1. State precisely the steps in your construction above.

2. Earlier in this lesson, we explained why the perpendicular bisectors are always concurrent. Using similar reasoning, explain clearly why the angle bisectors are always concurrent at the incenter.

3. Observe the constructions below. Point A is the ___________________________ of triangle $\triangle JKL$ (notice that it can fall outside of the triangle). Point B is the ___________________________ of triangle $\triangle RST$. The circumcenter of a triangle is the center of the circle that circumscribes that triangle. The incenter of the triangle is the center of the circle that is inscribed in that triangle.



On a separate piece of paper, draw two triangles of your own below and demonstrate how the circumcenter and incenter have these special relationships.

4. How can you use what you have learned in Problem 3 to find the center of a circle if the center is not shown?

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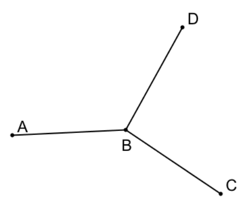
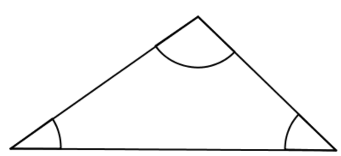
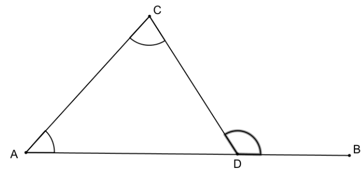
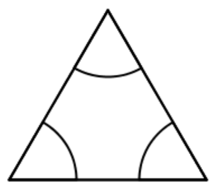
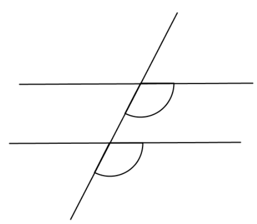
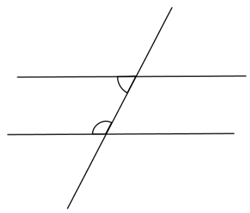
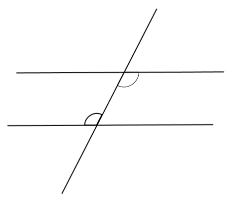
Problem Set

In previous years, you have studied many facts and made many discoveries about angles. Complete the chart below as a review of those facts and discoveries.

Fact/Discovery	Diagram	Abbreviation
Vertical angles are equal in measure.		vert. ∠s
Two angles that form a linear pair are supplementary.		∠s on a line
	 $\angle ABC + \angle CBD + \angle DBA = 360°$	∠s at a point
The sum of the 3 angle measures of any triangle is ______ .		∠ sum of Δ
When one angle of a triangle is a right angle, the sum of the measures of the other two angles is 90˚.		∠ sum of rt. Δ

		ext. $\angle$ of Δ
		base $\angle$s of isos. Δ
		equilat. Δ
		corr. $\angle$s, $\overline{AB} \parallel \overline{CD}$
If a transversal intersects two lines such that the measures of the corresponding angles are equal, then the lines are parallel.		corr. $\angle$s converse

If a transversal intersects two parallel lines, then the interior angles on the same side of the transversal are supplementary.		int. ∠s, $\overline{AB} \parallel \overline{CD}$
		int. ∠s converse
		alt. ∠s, $\overline{AB} \parallel \overline{CD}$
If a transversal intersects two lines such that measures of the alternate interior angles are equal, then the lines are parallel.		alt. ∠s converse